STUDY OF HEAT TRANSFER BETWEEN THERMOREACTIVE MIXTURES AND HEATED SURFACES

I. Ya. Bitsyutko, G. I. Bobryakov, V. P. Kozhin and G. A. Fateev

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Unsteady state heat transfer between a highly thermally conductive body and thermoreactive mixtures is analyzed. A method is presented for determining the effective thermophysical temperature profiles in a sphere cooling in the test mixture.

Mixtures of sand and thermosetting resin are used in the production of founder bars (castings). During production these mixtures are blown pneumatically into preheated (180-300°C) mould to fill it completely in a short time of a few seconds prior to solidification.

This paper was written because improvements in production technology require assessment of energy consumption. It also gives profiles for estimation of the development of the temperature profiles needed in the determination for solidification kinetics. Solidification results from several simultaneous physical and chemical processes which are not yet adequately understood. An idealized representation of the production process was studied with the heat transfer parameters having known values. The thermophysical parameters were determined under conditions similar to the real process, i. e. conditions of thermal shock (near instantaneous filling of the mould to expose the thermoreactive mixture very rapidly to heating) before the heating penetrates deeply through the volume (in production the heating period is brief).

The simulation analyzed was as follows. An isothermally hot body (a plate or sphere) was placed in contact with the test mixture. The resulting heat transfer was represented by the conduction equations with constant coefficients

$$\frac{\partial\Theta(X,\ T)}{\partial T}=\frac{\partial^2\Theta(X,\ T)}{\partial X^2}, \tag{1}$$

where

$$\Theta=\frac{t-t_\infty}{t_0-t_\infty};\quad X=\frac{c_2\rho_2 x}{c_1\rho_1 l};\quad T=\frac{\lambda_2 c_2\rho_2\tau}{(c_1\rho_1 l)^2}\quad \text{for a plate}$$

$$\Theta=\frac{c_2\rho_2 r(t-t_\infty)}{c_1\rho_1 r_0(t_0-t_\infty)};\quad X=\frac{c_2\rho_2 r}{c_1\rho_1 r_0};\quad T=\frac{\lambda_2 c_2\rho_2\tau}{(c_1\rho_1 r_0)^2}\quad \text{for a sphere.} \tag{2}$$

For the initial condition (T = 0) where the thermal shock occurs (contact of the hot body with the cold mixture):

$$\Theta(0,\ 0)=1,\quad \Theta(X,\ 0)=0\quad \text{for a plate}$$

$$\Theta(K,\ 0)=K,\quad \Theta(X,\ 0)=0\quad \text{for a sphere.} \tag{3}$$

At the boundary (the heat balance between the phases):

$$\frac{\partial\Theta(0,\ T)}{\partial T}=\frac{\partial\Theta(0,\ T)}{\partial X}\quad \text{for a plate}$$

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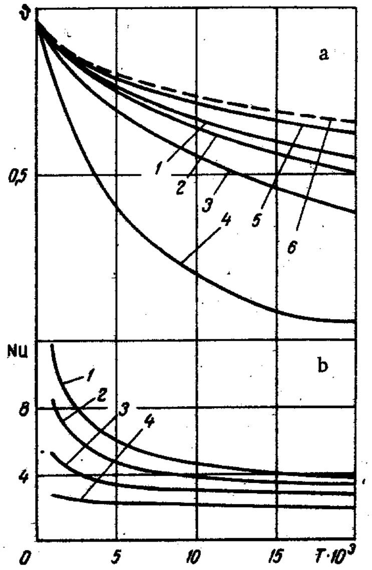

Fig. 1. Temperature of body and Nusselt number versus time, 1-5) sphere; 6) plate).

$$\frac{\partial \Theta (K,\ T)}{\partial T} = 3 \left[\frac{\partial \Theta (K,\ T)}{\partial X} - \frac{\Theta (K,\ T)}{K} \right] \quad \text{for a sphere}. \tag{4}$$

For the terminal condition

$$\Theta (\infty,\ T) = 0. \tag{5}$$

Manipulation gave the following solutions for a plate

$$\Theta (X,\ T) = \exp (X + T) \operatorname{erfc} \left(\sqrt{T} + \frac{X}{2\sqrt{T}} \right); \tag{6}$$

for a sphere

$$\Theta (X,\ T) = \int_0^T \Theta (K,\ \overline{T}) \cdot F (X,\ T - \overline{T})\, d\overline{T}, \tag{7}$$

where

$$F (X,\ T) = \frac{X - K}{2\sqrt{\pi T^3}} \exp \left[- \frac{(X - K)^2}{4T} \right]; \tag{8}$$

$$\Theta (K,\ T) = K \left[\frac{k_1^2 + k_1 k_2}{k_1^2 - k_2^2} \exp (k_1^2 T) \operatorname{erfc} (k_1 \sqrt{T}) - \frac{k_2^2 + k_1 k_2}{k_1^2 - k_2^2} \exp (k_2^2 T) \operatorname{erfc} (k_2 \sqrt{T}) \right], \tag{9}$$

when $(3/4)K > 1$;

$$\Theta (K,\ T) = K \left[(1 + 2\alpha^2 T) \exp (\alpha^2 T) \operatorname{erfc} (\alpha \sqrt{T}) - 2\alpha \sqrt{\frac{T}{\pi}} \right], \tag{10}$$

when $(3/4)K = 1$;

$$\Theta (K,\ T) = K \Big\{ \operatorname{erfc} (\alpha \sqrt{T}) \exp [(\alpha^2 - \beta^2) T] \times \left(\frac{\alpha}{\beta} \sin 2\alpha\beta T + \cos 2\alpha\beta T \right) - \frac{1}{\beta} \sqrt{\frac{T}{\pi}} \int_0^\beta \exp [-T (\beta^2 - y^2)] \times [2\alpha \cos 2\alpha T (\beta - y) - 2\beta \sin 2\alpha T (\beta - y)]\, dy \Big\}, \tag{11}$$

when $(3/4)K < 1$.

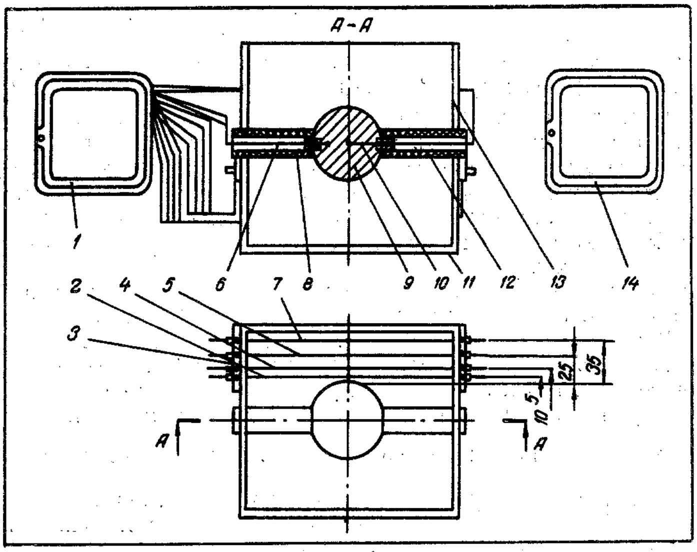

Fig. 2. Experimental installation: 1, 14) potentiometers; 2, 4, 5, 6, 7, 10) thermocouples; 3) coupling shoe; 8) bush; 9) sphere; 11) lower box; 12) hollow rod; 13) upper box.

In these equations

$$k_{1,2} = \frac{3}{2} \pm \sqrt{\frac{3}{K}\left(\frac{3}{4}K - 1\right)}, \quad \alpha = \frac{3}{2}, \quad \beta = \sqrt{\frac{3}{K}\left(1 - \frac{3}{4}K\right)}.$$

These solutions formed the basis of the present method of determining the effective thermophysical coefficients. In practice it is difficult to measure the temperature of the thermoreactive mixture but the thermophysical coefficients can nevertheless be obtained from the cooling temperature profiles of the heated body in the mixture. The temperature changes in the hot body during cooling provide sufficient data for specifying the heat transfer, as described by the above equations but in the reverse direction. Plotting the temperature against time for the simulation model provides information about temperature/time variation at the interface (a boundary condition of the first type) and the heat flux into the mixture (a boundary condition of the second type (4)).

Hence, consider the solution for body surface only (equation (6) when X = 0 for a plate and equations (9)-(10) for a sphere). From the cooling profile it is possible to determine the thermal coefficient $\varepsilon = \sqrt{\lambda_2 \rho_2 c_2}$, which forms the non-dimensionalized time since equation (6) for X = 0 contains only T as a variable. The cooling sphere profile, unlike that for a plate, alloys the two independent coefficients ε and λ to be determined since K occurs in the equation. For this reason experiments were made with spheres.

Although the principle of the method for determining ε and λ is presented above, it is necessary to show the validity of its application for the reverse direction of transfer. In doing so it is of interest to notice the dual nature of the sphere cooling graph (Fig. 1a) with different behavior of the curves for small and large values of T. For $T \to 0$ and $K \to \infty$ e.g. when the sphere radius r_0 is large compared with its scale (equivalent penetration depths $\varepsilon\sqrt{\tau}/\rho_1 c_1$) the sphere approximates to a plate with a cooling curve dependent on the parameter ε. Further, when $T \to 0$ the relationship characterizing the thermal shock [2] is

$$\vartheta = \frac{1}{2}\sqrt{\frac{T}{\pi}}. \tag{12}$$

As shown in Fig. 1a when $K \to 0$ the difference between the relationships for the sphere and plate increases even though the initial parts of the graphs may suggest that they are the same. The situation apparently precludes the possibility of determining ε from the experimental data. This can however be seen not to be so by noting that the nondimensional time $(\rho_1 c_1 r_0)^2/\lambda_2 \rho_2 c_2$ contains the same variables as K. If decrease in K occurs with changes in $\rho_2 c_2$ then a transformation will be observed with plotting on axis of ϑ and T/K, and similarly with increase in $\rho_1 c_1$ with plotting on axis of ϑ and T/K^2. This substantially decreases the nondimensional time and straightens the initial part of the curve dependent on the parameter ε and different from Fig. 1a when $K \to 0$.

TABLE 1. Effective Values of Thermophysical Transfer Parameters

Thermophysical constants	Test mixture				
	dry sand	wet sand		wet mixture	
		φ=2 %	φ=5 %	KΦ-90	K-27
ε, $W\cdot sec^{1/2}/m^2 \cdot deg$	504,1	543,4	587,9	652,4	682,1
λ, W/m· deg	0,256	0,685	1,099	0,826	0,805

In accord with the analysis Fig. 1b shows that after the initial unsteady state (the thermal shock) a second stage develops and can be characterized by an exponential cooling curve

$$\mathrm{Nu} = \frac{2}{3} K \frac{\partial \Theta}{\partial T} \Big/ \Theta \approx \text{const}, \quad (13)$$

This is called the steady state. For small values of K, the Nusselt number rapidly approaches 2 which is representative of a steady heat transfer from a sphere to an infinite medium. (It is the unsteady state of the transfer which causes the Nu = 2 to change.) The cooling rate found the exponential part of the experimental curve allows the value of λ to be determined as the only unknown in the Nusselt number.

This analysis leads to the conclusion of practical importance. The two stage character of sphere cooling curve allows the determination of two independent thermophysical parameters and it also enables the cooling process to be controlled by changing K.

Figure 2 shows the apparatus used. A copper sphere of 50 mm diameter with thermocouples near its centre and surface was heated to 250°C (this is the temperature used in production practice) and is placed in a wooden case. This case was then rapidly filled with the mixture being tested. To study the temperature in the vicinity of the sphere, thermocouples were placed at various distances from its surface. An ÉPP-09 potentiometer was used to record the temperature curves. Dry and wet sand mixed with KF-90 and KF-27 resin was used.

The experiments gave the rapid cooling of the sphere (particularly during the initial stage) and varying temperatures in its vicinity. As expected no differences were detected between readings of the thermocouples inside the sphere.

An algorithm was constructed to determine the two transfer parameters from the experimental results. The procedure for the algorithm was to choose the ε and λ such that the best agreement between the theoretical equation (11) and the experimental data was obtained (to minimize the mean square deviation between these data and theory). This calculation was done by the small difference method by computer [3].

The results for the thermophysical coefficients are given in Table 1. As shown in Fig. 3 the two parameter theoretical curve is in good agreement with the experimental points for the sphere cooling in all cases. The figure clearly shows the enhancement of cooling by even small moisture levels in the sand. The data corresponding to production mixtures appear amongst the curves for wet sand. Hence it may be assumed that the controlling factor in practice is the moisture initially contained in the resin and then evaporated during solidification. The enhancement of heat transfer can be represented quantitatively by the increases in the transfer and thermal conductivity coefficient above those with dry sand.

With dry sand the thermophysical coefficients are close to their normal values which control heat transfer. The heat transfer coefficients then obtained satisfactorily describe the temperature field in the vicinity of the sphere. Figure 3 shows the comparison between values measured for a distance of 5 mm from the sphere and calculated from equation (7).

With wet sand and production mixtures the heat transfer is more complex. The effect of increased thermal conductivity in moist media is well known [4]. The present values (Table 1) for heat transfer are however even bigger because of the evaporation at higher temperatures. Consequently effective values were obtained using the simplified physical simulation which cannot accurately represent the temperature

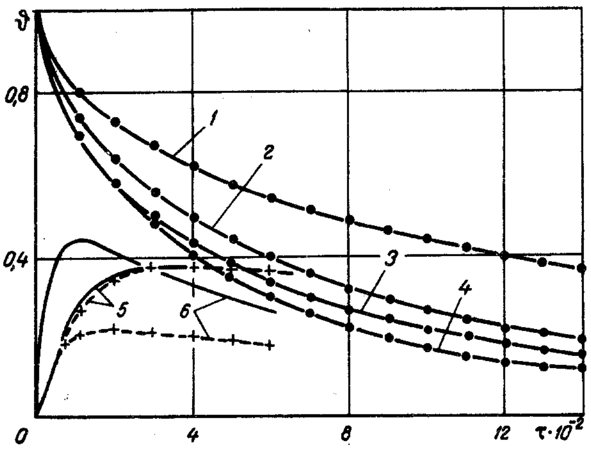

Fig. 3. Relative temperature versus time τ (sec) (solid lines, calculation; dots and dashed lines, experiment). On sphere surface: 1) dry sand; 2) moist sand, $\varphi = 2\%$; 4) moist sand, $\varphi = 5\%$. At a distance of 5 mm from sphere surface: 5) dry sand; 6) mixture on resin K-27 basis.

field in the vicinity of the sphere which is controlled by the thermal conductivity $a = \lambda^2/\varepsilon^2$. This is clearly shown by Figure 3 for K-27 production mixtures. As established above, the two parameters determined controling the cooling of the sphere vary with time as the process progresses. This therefore prevents their being used for calculating other dependent parameters specifying the development of the temperature field.

The effective values of λ and ε can however be used for engineering purposes, for estimating the heat transfer from a metal mould to casting mixtures. This is confirmed by the good correlation of the experimental data for the cooling sphere. A second conclusion from the agreement between the experimental graph and the two parameter theoretical analysis for the sphere cooling is that with careful work it is possible to determine the two transfer parameters separately from the temperature field in the vicinity of the sphere.

This analysis shows that the simulation used for conduction without phase changes allows calculation of efficiency and energy consumption of foundry casting equipment. Solution for the kinetics of solidification which controls the rate of solid growth can be obtained only using an algorithm allowing for transition and energy of phase changes.

NOTATION

c	is the heat capacity;
l	is the thickness of plate;
r	is the spherical coordinate;
t	is the temperature;
$K = \rho_2 c_2/\rho_1 c_1$	is the dimensionless coefficient;
λ	is the thermal conductivity;
ρ	is the density
τ	is the time;
$\vartheta = t - t_\infty / t_0 - t_\infty$	is the dimensionless temperature
φ	is the relative humidity.

Subscripts

0	denotes the wall conditions;
∞	denotes the infinity conditions;
1, 2	denotes the parameters in body and in medium.

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